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OPTIMAL SOLUTION OF FUZZY TRANSPORTATION PROBLEMS USING RANKING FUNCTION

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ABSTRACT

In real-world problems, optimization techniques are useful for solving problems like project schedules, assignment problems, and network flow analysis. This paper presents fuzzy transportation problems in which direct fuzzy costs, fuzzy supplies, and fuzzy demands transported quantities are fuzzy in nature. The objective is to minimize the total fuzzy cost under fuzzy decision variables.

Here, we are proposing an alternative method for solving the fuzzy transportation problem, where fuzzy demand and supply all are in the form of triangular fuzzy numbers. Where fuzzy demand and supply all are in the form of triangular fuzzy numbers. This method along with an illustrative numerical example to get the solution to the triangular fuzzy transportation problem having m origins and n destinations is discussed makers.

***KEYWORDS:** Triangular Fuzzy Numbers, Fuzzy Transportation Problem & Ranking Technique*

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Original Article

1. INTRODUCTION

The Transportation problem is a special type of linear programming problem which deals with the distribution of the single product (raw or finished) from various sources of supply to the various destination of demand in such a way that the total transportation cost is minimized. There are efficient algorithms for solving the transportation problems when all the decision parameters, i.e. the supply available at each source, the demand required at each destination as well as the unit transportation costs are given in a precise way. But in real life, there are many diverse situations due to uncertainty in one or more decision parameters and hence they may not be expressed in a precise way. This is due to measurement inaccuracy, lack of evidence, computational errors, high information cost, weather conditions etc. Hence we cannot apply the traditional classical methods to solve the transportation problems successfully. Therefore the use of Fuzzy transportation problems is more appropriate to model and solve the real world problems. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply, and demand are fuzzy quantities.

Bellman and Zadeh [1] proposed the concept of decision making in the Fuzzy environment. Chanas. et.al., [2] proposed the concept of the optimal solution for the Transportation with Fuzzy coefficient expressed as Fuzzy numbers. Srinivasan [6] - [11] described the new methods to solve the fuzzy transportation problem.

In this paper, An alternative method for solving a special type of fuzzy transportation problems. In this method transportation cost represented as triangular fuzzy numbers. The method is to rank the fuzzy objective values of the objective function by some ranking method for numbers to find the best alternative. On the basis of

this idea, the ranking method with the help of α solution has been adopted a transform the fuzzy transportation problem. To illustrate this method numerical example is solved and the obtained results are compared with the results of existing approaches. So the proposed approach is very easy to understand and to apply on real-life transportation problems for the decision makers.

2. TERMINOLOGY

In this section, some basic definitions of the fuzzy set theory are reviewed (Dubois and Prade, 1980), (Kauffman and Gupta, 1991).

2.1 Definition

The characteristic function $\mu_A(x)$ of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}(x)$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \to [0,1]$. The assigned value indicates the membership grade of the element in the set A. The function $\mu_{\tilde{A}}(x)$ is called the membership function and the set $\tilde{A} = \{(x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0,1]\}$. is called a fuzzy set.

2.2 Definition

A fuzzy set $\widetilde{A}$, defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_{\tilde{A}} : R \to [0,1]$ has the following characteristics

- $\widetilde{A}$ is normal. It means that there exists an x ∈ R such that $\mu_{\tilde{A}}(x) = 1$
- $\widetilde{A}$ is convex. It means that for every $x_1, x_2 \in R$, $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$, $\lambda \in [0,1]$
- $\mu_{\tilde{A}}$ is upper semi-continuous.
- Support ($\widetilde{A}$) is bounded in R.

2.3 Definition

A fuzzy number $\widetilde{A}$ is said to be a non-negative fuzzy number if and only $\mu_{\tilde{A}}(x) = 0, \ \forall \ x < 0$

2.4 Definition

A fuzzy number $\widetilde{A}$ in R is said to be a triangular fuzzy number if its membership function $\mu_{\tilde{A}} : R \to [0,1]$ has the following characteristics.

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ 1, & x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3 \\ 0, & otherwise \end{cases}$$

It is denoted by $\tilde{A} = (a^{(1)}, a^{(2)}, a^{(3)})$ where $a^{(1)}$ is Core ($\tilde{A}$), $a^{(2)}$ is left width and $a^{(3)}$ is right width. The geometric representation of the Triangular Fuzzy number is shown in the figure. Since the shape of the triangular fuzzy number $\tilde{A}$ is usually in the triangle it is called so.

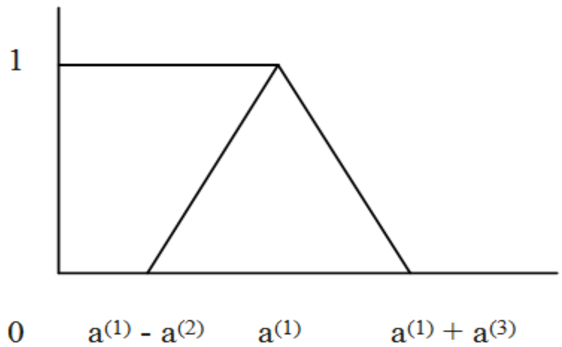

Figure 1: Membership Function of Triangular Fuzzy Number

The Parametric form of a triangular fuzzy number is represented by $\tilde{A} = \left[a^{(1)} - a^{(2)}(1-r),\ a^{(1)} + a^{(3)}(1-r)\right]$

2.5 Ranking of Triangular Fuzzy number

Several approaches for the ranking of fuzzy numbers have been proposed in the literature. An efficient approach for comparing the fuzzy numbers is by the use of a ranking function based on their graded means. That is, for every $\tilde{A} = (a^{(1)}, a^{(2)}, a^{(3)}) \in$ F(R), the ranking function $\Re$: F(R) $\to$ R by graded mean is defined as

$$\Re(\tilde{A}) = \left(\frac{a_1 + 4a_2 + a_3}{6}\right) (\because a_2 = a_3)$$

For any two fuzzy triangular Fuzzy numbers $\tilde{A} = (a^{(1)}, a^{(2)}, a^{(3)})$ and $\tilde{B} = (b^{(1)}, b^{(2)}, b^{(3)})$ in F(R), we have the following comparison

- $\tilde{A} < \tilde{B}$ If and only if $\Re(\tilde{A}) < \Re(\tilde{B})$
- $\tilde{A} > \tilde{B}$ If and only if $\Re(\tilde{A}) < \Re(\tilde{B})$
- $\tilde{A} \approx \tilde{B}$ If and only if $\Re(\tilde{A}) = \Re(\tilde{B})$
- $\tilde{A} - \tilde{B}$ If and only if $\Re(\tilde{A}) - \Re(\tilde{B}) = 0$

A triangular fuzzy number $\widetilde{A} = (a^{(1)}, a^{(2)}, a^{(3)})$ in F(R) is said to be positive if $\Re(\widetilde{A}) > 0$ and denoted by $\widetilde{A} > \tilde{0}$. Also if $\Re(\widetilde{A}) > 0$, then $\widetilde{A} > \tilde{0}$ and if $\Re(\widetilde{A}) = 0$, then $\widetilde{A} \approx \tilde{0}$. If $\Re(\widetilde{A}) = \Re(\widetilde{B})$, then the triangular numbers $\widetilde{A}$ and $\widetilde{B}$ are said to be equivalent and is denoted by $\widetilde{A} \approx \widetilde{B}$.

3. MATHEMATICAL FORMULATION OF A FUZZY TRANSPORTATION PROBLEM

Mathematically a transportation problem can be stated as follows:

Minimize

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{1}$$

Subject to

$$\left.\begin{aligned} &\sum_{j=1}^{n} x_{ij} = a_i \quad && j = 1,2,...,\mathrm{n} \\ &\sum_{i=1}^{m} x_{ij} = b_j \quad && i = 1,2,...,\mathrm{m} \\ &x_{ij} \geq 0 \quad && i = 1,2,...,m,\ j = 1,2,...,n \end{aligned}\right\} \tag{2}$$

Where c_{ij} is the cost of transportation of an unit from the ith source to the jth destination, and the quantity x_{ij} is to be some positive integer or zero, which is to be transported from the ith origin to jth destination. A obvious necessary and sufficient condition for the linear programming problem given in (1) to have a solution is that

$$\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} b_j \tag{3}$$

(i.e) assume that the total available is equal to the total required. If it is not true, a fictitious source or destination can be added. It should be noted that the problem has a feasible solution if and only if the condition (2) satisfied. Now, the problem is to determines x_{ij}, in such a way that the total transportation cost is minimum

Mathematically a fuzzy transportation problem can be stated as follows:

Minimize

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{4}$$

Subject to

$$\left.\begin{array}{ll} \sum_{j=1}^{n} x_{ij} = \tilde{a}_i & j=1,2,...,n \\ \sum_{i=1}^{m} x_{ij} = \tilde{b}_j & i=1,2,...,m \\ x_{ij} \geq 0 & i=1,2,...,m, \quad j=1,2,...,n \end{array}\right\} \tag{5}$$

In which the transportation costs $\tilde{c}_{ij}$, supply $\tilde{a}_i$ and demand $\tilde{b}_j$ quantities are fuzzy quantities. An obvious necessary and sufficient condition for the fuzzy linear programming problem give in (4-5) to have a solution is that

$$\sum_{i=1}^{n} \tilde{a}_i \simeq \sum_{j=1}^{m} \tilde{b}_j \tag{6}$$

This problem can also be represented as follows:

Table 1

	1	………	n	Supply
1	$\tilde{c}_{11}$	………	$\tilde{c}_{1n}$	$\tilde{a}_1$
. . . .	. . . .	. . . .	. . . .	. . . .
m	$\tilde{c}_{m1}$	……...	$\tilde{c}_{mn}$	$\tilde{a}_m$
Demand	$\tilde{b}_1$	………	$\tilde{b}_n$	

4. AN ALTERNATIVE METHOD FOR SOLVING TRANSPORTATION PROBLEM

Following are the steps for solving Transportation Problem

Step 1: Check whether the Transportation problem is balanced or not. If it is balanced then go to next step.

Step 2: From the given Transportation problem, convert fuzzy values to crisp values using the ranking function.

Step 3: Find the Harmonic mean for each row and column

Step 4: Discover the row/column with the maximum Harmonic mean value and select the cell with minimum cost in the resultant row or column.

Step 5: Maximum assign is made to a cell having minimum cost value. Delete that row/column where supply/demand exhausted.

Step 6: Repeat the step 5 to step 7 until all the demand and supply are fulfilled.

Step 7: Calculate the total cost

Total Cost = $\sum\sum C_{ij} X_{ij}$

5. NUMERICAL EXAMPLE

A company has four sources O_1, O_2, O_3 and O_4 and four destinations D_1, D_2, D_3 and D_4, the fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination is C_{ij} where

$[C_{ij}]_{3\times 3} = \begin{pmatrix} (4,5,6) & (3,4,5) & (6,7,8) \\ (1,2,3) & (5,6,7) & (4,5,6) \\ (3,4,5) & (7,8,9) & (2,3,4) \end{pmatrix}$ and fuzzy availability of the product at source is ((3,4,5) (5,6,7) (4,5,6)) and the fuzzy demand for the product at destinations are ((4,5,6) (5,6,7) (3,4,5)) respectively. The fuzzy transportation problems are

Table 2

	FD_1	FD_2	FD_3	Fuzzy Capacity
FO_1	[4,5,6]	[3,4,5]	[6,7,8]	[3,4,5]
FO_2	[1,2,3]	[5,6,7]	[4,5,6]	[5,6,7]
FO_3	[3,4,5]	[7,8,9]	[2,3,4]	[4,5,6]
Fuzzy Demand	[4,5,6]	[5,6,7]	[3,4,5]	

Solution

In Conformation to model the fuzzy transportation problem can be formulated in the following mathematical programming form

Min Z = $R(4,5,6)x_{11} + R(3,4,5)x_{12} + R(6,7,8)x_{13} + R(1,2,3)x_{21} + R(5,6,7)x_{22} + R(4,5,6)x_{23} + R(3,4,5)x_{31} + R(7,8,9)x_{32} + R(2,3,4)x_{33}$

$$R(4,5,6) = \frac{4+4*5+6}{6} = 5$$

Similarly

$$\begin{array}{lll} & R(3,4,5)=4; & R(6,7,8)=7 \\ R(1,2,3)=2; & R(5,6,7)=6; & R(4,5,6)=5 \\ R(3,4,5)=4; & R(7,8,9)=8; & R(2,3,4)=3 \end{array}$$

Rank of all supply

$$R(3,4,5)=4; \quad R(5,6,7)=6; \quad R(4,5,6)=5$$

Rank of all demand

$$R(4,5,6)=5; \quad R(5,6,7)=6; \quad R(3,4,5)=4$$

Table After Ranking

Table 3

	D1	D2	D3	Supply
O1	5	4	7	4
O2	2	6	5	6
O3	4	8	3	5
Demand	5	6	4	

With the help of the proposed method, we can get the below solution.

Table 4

	D1	**D2**	**D3**	**Supply**
O1	5	**4** 4	7	4
O2	**4** 2	**2** 6	5	6
O3	**1** 4	8	**4** 3	5
Demand	5	6	4	

Hence (3+3-1)=5 cells are allocated and hence we got our feasible soln. Next, we calculate total cost and its corresponding allocated value of supply-demand which is shown in Table - 4

Total Cost = (4x4)+(4x2)+(2x6)+(1x4)+(4x3)=52

This is a basic feasible solution. The solution obtained using NCM, LCM, VAM, and MODI/Stepping stone methods respectively. Hence the basic feasible solution obtained from the new method is optional soln.

6. CONCLUSIONS

In this paper, the transportation costs are considered as imprecise numbers described by fuzzy numbers which are more realistic and general in nature. Moreover, the fuzzy transportation problem of triangular fuzzy numbers has been transformed into crisp transportation problem using ranking indices. Numerical examples show that by this method we can have the optimal solution as well as the crisp and fuzzy optimal total cost. By using the ranking method we have shown that the total cost obtained is optimal. Moreover, one can conclude that the solution of fuzzy problems can be obtained by our proposed method effectively. This technique can also be used in solving other types of problems like project schedules, assignment problems and network flow problems.

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